4. Find the greatest and least values of the following functions on the given closed interval:

(a) $f(x) = x - 2\sqrt{x}$ on $[0, 9]$;

(b) $f(x) = x^4 - 8x^2 + 2$ on $[-1, 3]$;

(c) $f(x) = e^x \ln x$ on $[1, 2]$.

① Find critical pt

② Compare value of $f(x)$ at end points and critical pts

(b) $f'(x) = 4x^3 - 16x$ exists everywhere

Step 1 $f'(x) = 0 \iff 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$4x(x-2)(x+2) = 0$

$x = \underbrace{0, \ 2}_{\text{Two critical pts.}}$ or -2 (rejected, not in $[-1,3]$)

Step 2

$f(0) = 2$

$f(2) = -14 \leftarrow$ min

$f(-1) = -5$

$f(3) = 11 \leftarrow$ max

max value $= 11$

min value $= -14$

Some basic integrals

$\boxed{\text{Don't forget } C}$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$
$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$
$$\sin A \sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$
$$\sin A \cos A = \frac{1}{2}\sin 2A$$
$$\cos^2 A = \frac{1}{2}(1+\cos 2A)$$
$$\sin^2 A = \frac{1}{2}(1-\cos 2A)$$

$$\int (af(x) + bg(x))\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

$$\int k\,dx = kx + c$$

$$\int x^k\,dx = \frac{1}{k+1}x^{k+1} + C \quad \text{for } k \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \cos x\,dx = \sin x + C \qquad \int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

t-substitution

Let $t = \tan\frac{x}{2}$. Then

$$\tan x = \frac{2t}{1-t^2} \qquad dx = \frac{2}{1+t^2}\,dt$$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \cot x\,dx = \ln|\sin x| + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\,dx = -\ln|\csc x + \cot x| + C \qquad \int \frac{1}{1+x^2}\,dx = \arctan x + C$$

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2017)
Pre-coursework Exercise 6

Partial Fractions

1. Resolve the following expressions into partial fractions.

   (a) $\dfrac{5}{x^2+x-6}$ (Hint: $\dfrac{5}{x^2+x-6} \equiv \dfrac{A}{x+3} + \dfrac{B}{x-2}$)

   (b) $\dfrac{1}{x(x^2+1)}$ (Hint: $\dfrac{1}{x(x^2+1)} \equiv \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$)

   (c) $\dfrac{5x^2-3x+4}{(x+1)(x^2-2x+6)}$

2. Resolve the following expressions into partial fractions.

   (a) $\dfrac{x^2+3x}{x^2+3x+2}$

   (b) $\dfrac{x^4+2x+4}{(2x^2+3)(x-2)}$

   (c) $\dfrac{2x^5}{(x^2-1)(x^2-4)}$

3. Resolve the following expressions into partial fractions.

   (a) $\dfrac{x^3+1}{(x-2)^4}$ (Hint: $\dfrac{x^3+1}{(x-2)^4} \equiv \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-2)^3} + \dfrac{D}{(x-2)^4}$)

   (b) $\dfrac{2x^2+1}{x^2(x^2+1)^2}$ (Hint: $\dfrac{2x^2+1}{x^2(x^2+1)^2} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1} + \dfrac{Ex+F}{(x^2+1)^2}$)

Indefinite Integration

4. (a) Resolve $\dfrac{x^5+3x^2+1}{(x-1)(x^2+4)}$ into partial fractions.

   (b) Hence, evaluate $\displaystyle\int \frac{x^5+3x^2+1}{(x-1)(x^2+4)}\,dx$

5. **(Integration by substitutions)**
   Evaluate the following integrals.

   (a) $\displaystyle\int (2x-1)^{10}\,dx$

   (b) $\displaystyle\int \frac{1}{\sqrt{5x+7}}\,dx$

   (c) $\displaystyle\int \frac{x}{\sqrt{1+x^2}}\,dx$

   (d) $\displaystyle\int x^2\sqrt{x^3+2}\,dx$

(e) $\displaystyle\int e^x \sin e^x \, dx$

(f) $\displaystyle\int \frac{(\ln x)^4}{x} \, dx$

(g) $\displaystyle\int \frac{\cos x}{\sqrt{\sin^3 x}} \, dx$

(h) $\displaystyle\int \frac{1}{\sqrt{x} + \sqrt{x+1}} \, dx$

(i) $\displaystyle\int \frac{e^{3x}+1}{e^x+1} \, dx$

(j) $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} \, dx$

(k) $\displaystyle\int \sec 2x \tan 2x \, dx$

(l) $\displaystyle\int \left(1 - \cos\frac{x}{2}\right)^2 \sin\frac{x}{2} \, dx$

(m) $\displaystyle\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) \, dx$

6. **(Integration by parts)**
Evaluate the following integrals by using integration by parts.

(a) $\displaystyle\int x \sin\frac{x}{2} \, dx$

(b) $\displaystyle\int x \ln x \, dx$

(c) $\displaystyle\int x e^{3x} \, dx$

(d) $\displaystyle\int \tan^{-1} x \, dx$

(e) $\displaystyle\int \sin^{-1} x \, dx$

(f) $\displaystyle\int x \sec^2 x \, dx$

(g) $\displaystyle\int x^3 e^x \, dx$

(h) $\displaystyle\int e^x \sin x \, dx$

(i) $\displaystyle\int e^{-x} \cos x \, dx$

7. **(Powers of trigonometric functions)**
Evaluate the following integrals.

(a) $\displaystyle\int \cos^3 x \sin x \, dx$

(b) $\displaystyle\int \sin^4 x \cos x \, dx$

(c) $\displaystyle\int \sin^3 x \, dx$

(d) $\displaystyle\int \cos^3 x \, dx$

(e) $\displaystyle\int \cos^4 x \sin^2 x \, dx$

(f) $\displaystyle\int \sec^2 x \tan x \, dx$

(g) $\displaystyle\int \sec^3 x \tan x \, dx$

(h) $\displaystyle\int \sec^4 x \tan^2 x \, dx$

8. **(Products of sines and cosines)**
Evaluate the following integrals.

(a) $\displaystyle\int \cos 3x \sin 2x \, dx$

(b) $\displaystyle\int \sin^3 x \sin 3x \, dx$

(c) $\displaystyle\int \cos x \cos 7x \, dx$

(d) $\displaystyle\int \sin^2 x \cos 3x \, dx$

(e) $\displaystyle\int \cos^3 x \sin 2x \, dx$

(f) $\displaystyle\int \sin x \sin 2x \sin 3x \, dx$

9. **(Trigonometric substitutions)**
Evaluate the following integrals.

(a) $\displaystyle\int \sqrt{25 - x^2} \, dx$

(b) $\displaystyle\int \frac{1}{8 + 2x^2} \, dx$

(c) $\displaystyle\int \frac{1}{\sqrt{4 + x^2}} \, dx$

(d) $\displaystyle\int \frac{x^2}{\sqrt{9 - x^2}} \, dx$

(e) $\displaystyle\int \frac{1}{x^2\sqrt{x^2 - 1}} \, dx$

(f) $\displaystyle\int \frac{x^2}{4 + x^2} \, dx$

(g) $\displaystyle\int \frac{2}{x^3\sqrt{x^2 - 1}} \, dx$

10. **(Integration of rational functions by partial fractions)**
Evaluate the following integrals.

(a) $\displaystyle\int \frac{x+4}{x^2 + 5x - 6} \, dx$

(b) $\displaystyle\int \frac{x+3}{2x^3 - 8x} \, dx$

(c) $\displaystyle\int \frac{x^3}{x^2 + 2x + 1} \, dx$

(d) $\displaystyle\int \frac{x^2}{(x-1)(x+1)^2} \, dx$

(e) $\displaystyle\int \frac{1}{(x+1)(x^2+1)} \, dx$

(f) $\displaystyle\int \frac{x^2}{x^4 - 1} \, dx$

(g) $\displaystyle\int \frac{x^4}{x^2 - 4} \, dx$

11. **(Integration by t-substitution)**

(a) Let $t = \tan\frac{x}{2}$, show that $\frac{dt}{dx} = \frac{1}{2}(1 + t^2)$.

(b) Express $\sin x$ and $\cos x$ in terms of t.

(c) By considering the substitution $t = \tan\frac{x}{2}$, evaluate the following integrals.

(i) $\displaystyle\int \frac{1}{2 + \sin x} \, dx$

(ii) $\displaystyle\int \frac{1}{3 - 2\cos x} \, dx$

(iii) $\displaystyle\int \frac{1}{2 + \sin x + \cos x} \, dx$

(iv) $\displaystyle\int \frac{1}{(2 + \cos x)\sin x} \, dx$

5. **(Integration by substitutions)**
Evaluate the following integrals.

(a) $\int (2x-1)^{10}\,dx$

(b) $\int \frac{1}{\sqrt{5x+7}}\,dx$

(c) $\int \frac{x}{\sqrt{1+x^2}}\,dx$

(d) $\int x^2\sqrt{x^3+2}\,dx$ $= \int \sqrt{x^3+2}\,\frac{1}{3}d(x^3+2)$

$= \frac{1}{3}\frac{2}{3}(x^3+2)^{\frac{3}{2}} + C$ same

no u

Let $u = x^3+2$ $du = 3x^2dx$

use u

$\int \frac{1}{3}u^{\frac{1}{2}}du = \frac{1}{3}\frac{2}{3}u^{\frac{3}{2}}+C$

or

$= \frac{2}{9}(x^3+2)^{\frac{3}{2}} + C$

(e) $\int e^x \sin e^x\,dx$ de^x

(f) $\int \frac{(\ln x)^4}{x}\,dx$ $\frac{1}{x}dx = d\ln x$

(g) $\int \frac{\cos x}{\sqrt{\sin^3 x}}\,dx$ $\cos x\,dx = d\sin x$

(h) $\int \frac{1}{\sqrt{x}+\sqrt{x+1}}\,dx$ rationalize

(i) $\int \frac{e^{3x}+1}{e^x+1}\,dx$

(j) $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

(k) $\int \sec 2x \tan 2x\,dx$ $= \frac{1}{2}\int \sec 2x \tan 2x\,d(2x)$

$= \frac{1}{2}\sec 2x + C$

(l) $\int \left(1-\cos\frac{x}{2}\right)^2 \sin\frac{x}{2}\,dx$

(m) $\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)dx$

$\int \sec\theta\tan\theta\,d\theta$

$= \sec\theta + C$

$\int \frac{1}{\sqrt{x}+\sqrt{x+1}} \cdot \frac{\sqrt{x}-\sqrt{x+1}}{\sqrt{x}-\sqrt{x+1}}\,dx$

$= \int \frac{\sqrt{x}-\sqrt{x+1}}{-1}\,dx$

(i) $\frac{e^{3x}+1}{e^x+1}$

$\shortparallel$

$e^{2x}-e^x+1$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

7. **(Powers of trigonometric functions)**
Evaluate the following integrals.

(a) $\int \cos^3 x \sin x \, dx$

(b) $\int \sin^4 x \boxed{\cos x \, dx = d\sin x}$

(c) $\int \sin^3 x \, dx$

(d) $\int \cos^3 x \, dx$

(e) $\int \cos^4 x \sin^2 x \, dx$

(f) ~~$\int \sec^2 x \tan x \, dx$~~

(g) ~~$\int \sec^3 x \tan x \, dx$~~

(h) ~~$\int \sec^4 x \tan^2 x \, dx$~~

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\sin A \sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$

$\sin A \cos A = \frac{1}{2}\sin 2A$

$\cos^2 A = \frac{1}{2}(1+\cos 2A)$

$\sin^2 A = \frac{1}{2}(1-\cos 2A)$

7b. $\int \sin^4 x \cos x \, dx = \int \sin^4 x \, d\sin x = \frac{1}{5}\sin^5 x + C$

$$\begin{aligned}\int \sin^4 x \cos^3 x \, dx &= \int \sin^4 x \cos^2 x \, \underline{\cos x \, dx} \\ &= \int \sin^4 x (1-\sin^2 x) \, d\sin x \\ &= \int (\sin^4 x - \sin^6 x) \, d\sin x \\ &= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C\end{aligned}$$

$\cos^2 x = 1 - \sin^2 x$

$$\begin{aligned}\int \sin^4 x \cos^5 x \, dx &= \int \sin^4 x \cos^4 x \, \underline{\cos x \, dx} \\ &= \int \underbrace{\sin^4 x (1-\sin^2 x)^2}_{\text{polynomial in } \sin x} \, d\sin x\end{aligned}$$

$$\begin{aligned}&\int \cos^4 x \sin^2 x \, dx \\ = &\int \frac{1}{4}(1+\cos 2x)^2 \, \frac{1}{2}(1-\cos 2x) \, dx \\ &\vdots\end{aligned}$$

See lecture note

8. **(Products of sines and cosines)**
Evaluate the following integrals.

(a) $\int \cos 3x \sin 2x \, dx$

(b) $\int \sin^3 x \sin 3x \, dx$

(c) $\int \cos x \cos 7x \, dx$

(d) $\int \sin^2 x \cos 3x \, dx$

(e) $\int \cos^3 x \sin 2x \, dx$

(f) $\int \sin x \sin 2x \sin 3x \, dx$

Different angles

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$$

$$\sin A \cos A = \frac{1}{2}\sin 2A$$

$$\cos^2 A = \frac{1}{2}(1+\cos 2A)$$

$$\sin^2 A = \frac{1}{2}(1-\cos 2A)$$

$$\frac{d5x}{dx} = 5$$

$$= \frac{1}{2}\int (\sin 5x - \sin x)\,dx$$

$$= \frac{1}{10}\int \sin 5x \, d5x - \frac{1}{2}\int \sin x dx$$

$$= \frac{1}{10}(-\cos 5x) + \frac{1}{2}\cos x + C$$

$$\int \cos^2 x \sin 2x \cos x dx$$

$$= \int \underbrace{\cos^2 x}_{1-\sin^2 x} \sin 2x \, d\sin x$$

10. **(Integration of rational functions by partial fractions)**
Evaluate the following integrals.

(a) $\displaystyle\int \frac{x+4}{x^2+5x-6}\,dx$

(b) $\displaystyle\int \frac{x+3}{2x^3-8x}\,dx$

(c) $\displaystyle\int \frac{x^3}{x^2+2x+1}\,dx$

(d) $\displaystyle\int \frac{x^2}{(x-1)(x+1)^2}\,dx$

(e) $\displaystyle\int \frac{1}{(x+1)(x^2+1)}\,dx$

(f) $\displaystyle\int \frac{x^2}{x^4-1}\,dx$

(g) $\displaystyle\int \frac{x^4}{x^2-4}\,dx$

$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$\int \frac{Bx+C}{x^2+1}dx = B\int \frac{x}{x^2+1}dx + C\int \frac{1}{x^2+1}dx$$

$$= \frac{B}{2}\int \frac{d(x^2+1)}{x^2+1} + C\arctan x$$

$$= \frac{B}{2}\ln|x^2+1| + C\arctan x + D$$

a. $x^2+5x-6 = (x+6)(x-1)$

Let $\dfrac{x+4}{x^2+5x-6} = \dfrac{A}{x+6} + \dfrac{B}{x-1}$

$x+4 = A(x-1) + B(x+6)$

$= (A+B)x + (-A+6B)$

$A+B=1$

$-A+6B=4$

$7B=5 \Rightarrow B=\frac{5}{7}$

$A=\frac{2}{7}$

$$\therefore \int \frac{x+4}{x^2+5x-6}dx = \frac{2}{7}\int\frac{1}{x+6}dx + \frac{5}{7}\int\frac{1}{x-1}dx$$

$$= \frac{2}{7}\ln|x+6| + \frac{5}{7}\ln|x-1| + C$$

c.

$$\begin{array}{r|l} & x-2 \\ \hline x^2+2x+1 & x^3 \\ & x^3+2x^2+x \\ \hline & -2x^2-x-0 \\ & -2x^2-4x-2 \\ \hline & 3x+2 \end{array}$$

$$\frac{x^3}{x^2+2x+1} = x-2+\underbrace{\frac{3x+2}{x^2+2x+1}}$$

$$\frac{A}{x+1} + \frac{B}{(x+1)^2}$$

9. **(Trigonometric substitutions)** Evaluate the following integrals.

$\int \sqrt{\text{quadratic}}$

(a) $\int \sqrt{25 - x^2}\, dx$

(b) $\int \frac{1}{8 + 2x^2}\, dx$

(c) $\int \frac{1}{\sqrt{4 + x^2}}\, dx$

(d) $\int \frac{x^2}{\sqrt{9 - x^2}}\, dx$

(e) $\int \frac{1}{x^2\sqrt{x^2 - 1}}\, dx$

(f) $\int \frac{x^2}{4 + x^2}\, dx$

(g) $\int \frac{2}{x^3\sqrt{x^2 - 1}}\, dx$

(h) $\int \frac{1}{\sqrt{5 - 4x - x^2}}$

$\sqrt{a^2 - x^2}$ $x = a\sin\theta$

$\sqrt{a^2 + x^2}$ $x = a\tan\theta$

$\sqrt{x^2 - a^2}$ $x = a\sec\theta$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

a. Let $x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$

$$\int \sqrt{25 - x^2}\, dx = \int \sqrt{25 - 25\sin^2\theta}\; 5\cos\theta\, d\theta$$
$$= \int \sqrt{25\cos^2\theta}\; 5\cos\theta\, d\theta$$
$$= 25\int \cos^2\theta\, d\theta$$
$$= \frac{25}{2}\int (1 + \cos 2\theta)\, d\theta$$
$$= \frac{25}{2}\theta + \frac{25}{4}\sin 2\theta + C$$
$$= \frac{25}{2}\arcsin\frac{x}{5} + \cdots$$

c. Let $x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{1}{\sqrt{4 + x^2}}\, dx = \int \frac{1}{\sqrt{4 + 4\tan^2\theta}}\; 2\sec^2\theta\, d\theta$$
$$= \int \frac{1}{2\sec\theta}\; 2\sec^2\theta\, d\theta$$
$$= \int \sec\theta\, d\theta$$
$$= \ln|\sec\theta + \tan\theta| + C$$

$$\int \frac{dx}{\sqrt{5-4x-x^2}} = \int \frac{dx}{\sqrt{9-4-4x-x^2}} = \int \frac{dx}{\sqrt{9-(x+2)^2}}$$

Let $x+2 = 3\sin\theta$ . . .

11. **(Integration by t-substitution)**

(a) Let $t = \tan\frac{x}{2}$, show that $\frac{dt}{dx} = \frac{1}{2}(1+t^2)$.

(b) Express $\sin x$ and $\cos x$ in terms of t.

(c) By considering the substitution $t = \tan\frac{x}{2}$, evaluate the following integrals.

(i) $\int \frac{1}{2+\sin x}\,dx$ (iii) $\int \frac{1}{2+\sin x+\cos x}\,dx$

(ii) $\int \frac{1}{3-2\cos x}\,dx$ (iv) $\int \frac{1}{(2+\cos x)\sin x}\,dx$

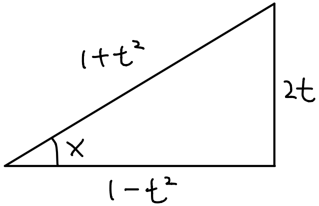
t-substitution

Let $t = \tan\frac{x}{2}$. Then

$$\tan x = \frac{2t}{1-t^2} \qquad dx = \frac{2}{1+t^2}\,dt$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$



$$\int \frac{1}{2+\sin x+\cos x}\,dx$$

$$= \int \frac{1}{2+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt$$

$$= \int \frac{2}{2+2t^2+2t+1-t^2}\,dt$$

$$= \int \frac{2\,dt}{t^2+2t+3} = \int \frac{2\,dt}{(t+1)^2+2}$$

$$= \int \frac{dt}{\left(\frac{t+1}{\sqrt{2}}\right)^2+1} = \int \frac{\sqrt{2}\,d\frac{t+1}{\sqrt{2}}}{\left(\frac{t+1}{\sqrt{2}}\right)^2+1} = \sqrt{2}\arctan\left(\frac{t+1}{\sqrt{2}}\right)+C$$

6. **(Integration by parts)** $\int u\,dv = uv - \int v\,du$ Product of

Evaluate the following integrals by using integration by parts.

e^x $\sin x$ $\cos x$

x^n

$\ln x$ $\arcsin x$ $\arccos x$

(a) $\displaystyle\int x \sin\frac{x}{2}\,dx$

(b) $\displaystyle\int x \ln x\,dx$

(c) $\displaystyle\int x e^{3x}\,dx$

(d) $\displaystyle\int \tan^{-1} x\,dx$

(e) $\displaystyle\int \sin^{-1} x\,dx$

Integration by parts

(f) $\displaystyle\int x \sec^2 x\,dx$ $= \int x\,d\tan x$

(g) $\displaystyle\int x^3 e^x\,dx$

(h) $\displaystyle\int e^x \sin x\,dx$

twice

(i) $\displaystyle\int e^{-x} \cos x\,dx$

e. $\int \sin^{-1} x\,dx$

$= x\sin^{-1}x - \int x\,d\sin^{-1}x$

$= x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$\vdots$

a. $\int \underset{\text{diff}}{x}\, \underbrace{\sin\frac{x}{2}}_{\text{int}}\,dx = 2\int x \sin\frac{x}{2}\,d\frac{x}{2}$

$= -2\int x\,d\cos\frac{x}{2}$

$= -2\left[x\cos\frac{x}{2} - \int \cos\frac{x}{2}\,dx\right]$

$= -2x\cos\frac{x}{2} + 2\int \cos\frac{x}{2}\,dx$

$= -2x\cos\frac{x}{2} + 4\sin\frac{x}{2} + C$

b. $\int x \ln x\,dx$

$= \frac{1}{2}\int \ln x\,dx^2$

$= \frac{1}{2}\left(x^2\ln x - \int x^2\,d\ln x\right)$

$= \frac{1}{2}\left(x^2\ln x - \int \frac{x^2}{x}\,dx\right)$

(h) $\int e^x \sin x\,dx$

$= \int \sin x\, de^x$ "integrate e^x"

"Integrate e^x twice"

$= e^x \sin x - \int e^x\, d\sin x$

$= e^x \sin x - \int e^x \cos x\,dx$

$= e^x \sin x - \int \cos x\, de^x$

$= e^x \sin x - \left[e^x \cos x - \int e^x\, d\cos x\right]$

$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$

$2\int e^x \sin x\,dx = e^x(\sin x - \cos x)$

$\int e^x \sin x\,dx = \frac{1}{2} e^x(\sin x - \cos x)$